Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Subsidiary

Thursday 15 January 2009 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (*a*) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2 \sqrt{(5x - 1)}.$$

(6)

(*b*) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

(4)

2.

$$f(x) = \frac{2x+2}{x^2 - 2x - 3} - \frac{x+1}{x-3}.$$

(*a*) Express f(x) as a single fraction in its simplest form.

(4)

(*b*) Hence show that $f'(x) = \frac{2}{(x-3)^2}$.

(3)

3.

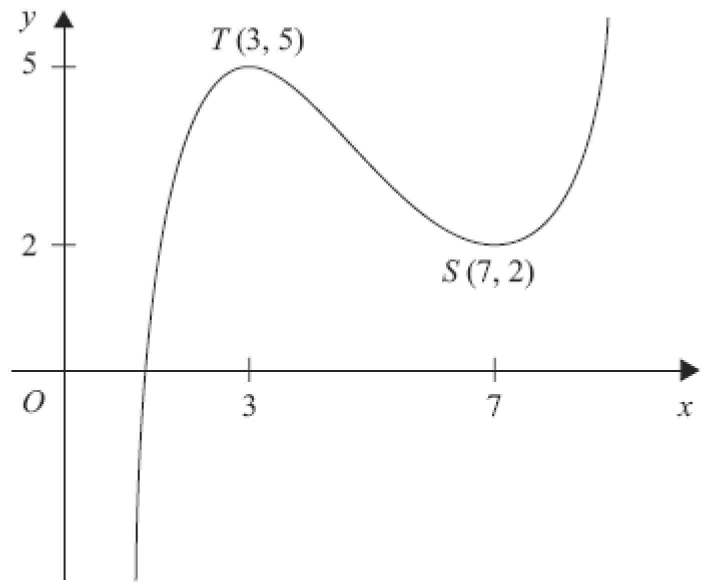

Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $1 < x < 9$.

The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(*a*) $y = 2\mathrm{f}(x) - 4$,

(3)

(*b*) $y = |\mathrm{f}(x)|$.

(3)

Indicate on each diagram the coordinates of any turning points on your sketch.

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4. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

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5. The functions f and g are defined by

$$f : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R},$$

$$g : x \mapsto e^{x^2}, \quad x \in \mathbb{R}.$$

(*a*) Write down the range of g.

(1)

(*b*) Show that the composite function fg is defined by

$$fg : x \mapsto x^2 + 3e^{x^2}, \quad x \in \mathbb{R}.$$

(2)

(*c*) Write down the range of fg.

(1)

(*d*) Solve the equation $\frac{d}{dx}\left[fg(x)\right] = x(xe^{x^2} + 2)$.

(6)

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6. (*a*) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(*b*) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

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7. $$f(x) = 3xe^x - 1.$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P.

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$.

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}.$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

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8. (a) Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of $3\cos\theta + 4\sin\theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3\cos(15t)° + 4\sin(15t)°,$$

where t is the time in hours from midday and $0 \leq t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)

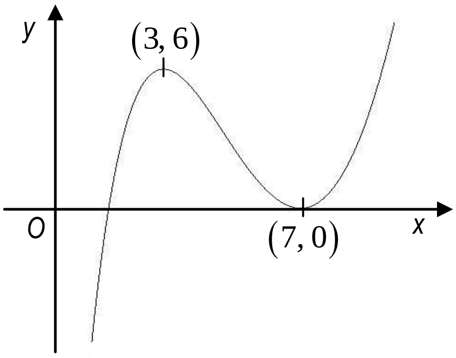
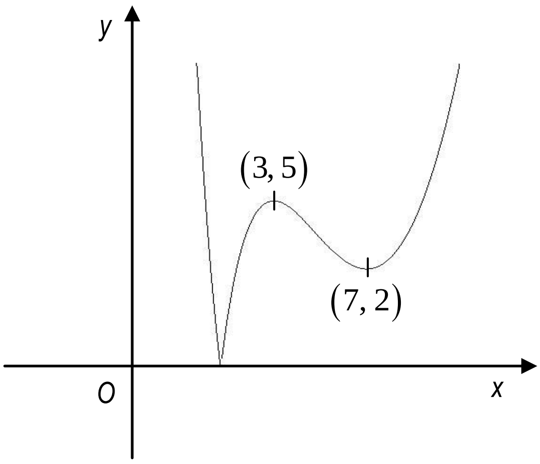
TOTAL FOR PAPER: 75 MARKS

END

January 2009
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{(5x-1)}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left((5x-1)^{\frac{1}{2}}\right)$	
	$=5\times\frac{1}{2}(5x-1)^{-\frac{1}{2}}$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x\sqrt{(5x-1)}+\frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$	M1 A1ft
	At $x=2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=4\sqrt{9}+\frac{10}{\sqrt{9}}=12+\frac{10}{3}$	M1
	$=\frac{46}{3}$ Accept awrt 15.3	A1 (6)
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sin 2x}{x^2}\right)=\frac{2x^2\cos 2x-2x\sin 2x}{x^4}$	M1 $\frac{\text{A1+A1}}{\text{A1}}$ (4) **[10]**
	Alternative to (b)	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin 2x\times x^{-2}\right)=2\cos 2x\times x^{-2}+\sin 2x\times(-2)x^{-3}$	M1 A1 + A1
	$=2x^{-2}\cos 2x-2x^{-3}\sin 2x \quad \left(=\frac{2\cos 2x}{x^2}-\frac{2\sin 2x}{x^3}\right)$	A1 (4)

Question Number	Scheme	Marks	
2 (a)	$\frac{2x+2}{x^2-2x-3}-\frac{x+1}{x-3}=\frac{2x+2}{(x-3)(x+1)}-\frac{x+1}{x-3}$		
	$=\frac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$	M1 A1	
	$=\frac{(x+1)(1-x)}{(x-3)(x+1)}$	M1	
	$=\frac{1-x}{x-3}$ Accept $-\frac{x-1}{x-3}$, $\frac{x-1}{3-x}$	A1	(4)
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1-x}{x-3}\right)=\frac{(x-3)(-1)-(1-x)1}{(x-3)^2}$	M1 A1	
	$=\frac{-x+3-1+x}{(x-3)^2}=\frac{2}{(x-3)^2}$ ∗ cso	A1	(3)
			[7]
	Alternative to (a)		
	$\frac{2x+2}{x^2-2x-3}=\frac{2(x+1)}{(x-3)(x+1)}=\frac{2}{x-3}$	M1 A1	
	$\frac{2}{x-3}-\frac{x+1}{x-3}=\frac{2-(x+1)}{x-3}$	M1	
	$=\frac{1-x}{x-3}$	A1	(4)
	Alternatives to (b)		
	① $\mathrm{f}(x)=\frac{1-x}{x-3}=-1-\frac{2}{x-3}=-1-2(x-3)^{-1}$		
	$\mathrm{f}'(x)=(-1)(-2)(x-3)^{-2}$	M1 A1	
	$=\frac{2}{(x-3)^2}$ ∗ cso	A1	(3)
	② $\mathrm{f}(x)=(1-x)(x-3)^{-1}$		
	$\mathrm{f}'(x)=(-1)(x-3)^{-1}+(1-x)(-1)(x-3)^{-2}$	M1	
	$=-\frac{1}{x-3}-\frac{1-x}{(x-3)^2}=\frac{-(x-3)-(1-x)}{(x-3)^2}$	A1	
	$=\frac{2}{(x-3)^2}$ ∗	A1	(3)

Question Number	Scheme		Marks	
3 (a)		Shape $(3, 6)$ $(7, 0)$	B1 B1 B1	 (3)
(b)		Shape $(3, 5)$ $(7, 2)$	B1 B1 B1	 (3) **[6]**

Question Number	Scheme		Marks
4	$x = \cos(2y + \pi)$		
	$\frac{dx}{dy} = -2\sin(2y + \pi)$		M1 A1
	$\frac{dy}{dx} = -\frac{1}{2\sin(2y + \pi)}$	Follow through their $\frac{dx}{dy}$ before or after substitution	A1ft
	At $y = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{1}{2\sin\frac{3\pi}{2}} = \frac{1}{2}$		B1
	$y - \frac{\pi}{4} = \frac{1}{2}x$		M1
	$y = \frac{1}{2}x + \frac{\pi}{4}$		A1 (6)
			[6]

Question Number	Scheme	Marks
5 (a)	$\mathrm{g}(x) \geq 1$	B1 (1)
(b)	$\mathrm{f\,g}(x) = \mathrm{f}\left(\mathrm{e}^{x^2}\right) = 3\mathrm{e}^{x^2} + \ln \mathrm{e}^{x^2}$	M1
	$= x^2 + 3\mathrm{e}^{x^2}$ ✱	A1 (2)
	$\left(\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}\right)$	
(c)	$\mathrm{fg}(x) \geq 3$	B1 (1)
(d)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 + 3\mathrm{e}^{x^2}\right) = 2x + 6x\,\mathrm{e}^{x^2}$	M1 A1
	$2x + 6x\,\mathrm{e}^{x^2} = x^2\,\mathrm{e}^{x^2} + 2x$	
	$\mathrm{e}^{x^2}\left(6x - x^2\right) = 0$	M1
	$\mathrm{e}^{x^2} \neq 0$, $\quad 6x - x^2 = 0$	A1
	$x = 0, 6$	A1 A1 (6)
		[10]

Question Number	Scheme	Marks
6 (a)(i)	$\sin 3\theta = \sin(2\theta+\theta)$	
	$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$	
	$= 2\sin\theta\cos\theta.\cos\theta + (1-2\sin^2\theta)\sin\theta$	M1 A1
	$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$	M1
	$= 3\sin\theta - 4\sin^3\theta$ ∗ cso	A1 (4)
(ii)	$8\sin^3\theta - 6\sin\theta + 1 = 0$	
	$-2\sin 3\theta + 1 = 0$	M1 A1
	$\sin 3\theta = \frac{1}{2}$	M1
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$	A1 A1 (5)
(b)	$\sin 15° = \sin(60° - 45°) = \sin 60°\cos 45° - \cos 60°\sin 45°$	M1
	$= \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} - \frac{1}{2}\times\frac{1}{\sqrt{2}}$	M1 A1
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ ∗ cso	A1 (4)
		[13]
	Alternatives to (b)	
	① $\sin 15° = \sin(45° - 30°) = \sin 45°\cos 30° - \cos 45°\sin 30°$	M1
	$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{2}$	M1 A1
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ ∗ cso	A1 (4)
	② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$	
	$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$	
	$\sin^2 15° = \frac{2-\sqrt{3}}{4}$	M1 A1
	$\left(\frac{1}{4}(\sqrt{6}-\sqrt{2})\right)^2 = \frac{1}{16}(6+2-2\sqrt{12}) = \frac{2-\sqrt{3}}{4}$	M1
	Hence $\sin 15° = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ ∗ cso	A1 (4)

Question Number	Scheme	Marks
7 (a)	$f'(x) = 3e^x + 3xe^x$	M1 A1
	$3e^x + 3xe^x = 3e^x(1+x) = 0$	
	$x = -1$	M1 A1
	$f(-1) = -3e^{-1} - 1$	B1 (5)
(b)	$x_1 = 0.2596$	B1
	$x_2 = 0.2571$	B1
	$x_3 = 0.2578$	B1 (3)
(c)	Choosing $(0.257\,55, 0.257\,65)$ or an appropriate tighter interval.	M1
	$f(0.257\,55) = -0.000\,379\,...$	
	$f(0.257\,65) = 0.000\,109\ \ldots$	A1
	Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ ✱ cso	A1
	($\Rightarrow x = 0.2576$, is correct to 4 decimal places)	(3)
		[11]
	Note: $x = 0.257\,627\,65\ \ldots$ is accurate	

Question Number	Scheme		Marks	
8 (a)	$R^2 = 3^2 + 4^2$		M1	
	$R = 5$		A1	
	$\tan\alpha = \frac{4}{3}$		M1	
	$\alpha = 53 \ldots ^\circ$	awrt 53°	A1	(4)
(b)	Maximum value is 5	ft their R	B1 ft	
	At the maximum, $\cos(\theta - \alpha) = 1$ *or* $\theta - \alpha = 0$		M1	
	$\theta = \alpha = 53 \ldots ^\circ$	ft their α	A1 ft	(3)
(c)	$\mathrm{f}(t) = 10 + 5\cos(15t - \alpha)^\circ$			
	Minimum occurs when $\cos(15t - \alpha)^\circ = -1$		M1	
	The minimum temperature is $(10 - 5)^\circ = 5^\circ$		A1 ft	(2)
(d)	$15t - \alpha = 180$		M1	
	$t = 15.5$	awrt 15.5	M1 A1	(3)
				[12]